OPTIMIZATION OF A FIRM'S CAPITAL STRUCTURE:
A QUANTITATIVE APPROACH BASED ON A PROBABILISTIC PROGNOSIS OF
RISK AND TIME OF BANKRUPTCY.

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Abstract.

The paper develops a new probabilistic approach to the problem of optimization of a firm’s capital structure.

The main idea of the approach is straightforward. As a possible firm’s bankruptcy is the principal factor restricting the amount of borrowed capital, we assess probabilities of a bankruptcy at various time horizons in future in dependence on the proportion of debt capital and other indices of a firm's current financial position and then calculate how these probabilities influence the firm’s value.

We identify a set of factors determining conditions of existence and value of the optimal debt / equity ratio. These include characteristics of a firm’s debt (proportion of short-term component of the debt, cost of service and maturity horizons of long-term component), characteristics of a firm’s ability to pay the debt, some macroeconomic factors.

We represent dependencies of optimal debt / equity ratio and gains in a firm’s value on the main influencing factors.

The approach is based on real data of real firms and does not use superfluously formalized models. We believe it can be used in practical capital structure decisions though specific calculations must be fulfilled for each firm that needs such decision.

JEL Codes: G32, G33.

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The problem of optimization of corporate capital structure is one of the central problems of corporate finance and has important applications for practical decision-making concerning financing of current operations and investment projects of corporations. The investigation of the problem was initialized by the seminal papers of Modigliani and Miller (1958, 1963), who studied the influence of tax advantage of debt financing on firms’ capital structure decisions. These papers as well as some papers appearing later (Scott (1976), Kraus, Litzenberger (1973), Baxter (1967), Brennan, Schwartz (1978) et al.) have formed the common opinion that optimal proportion of stockholders own and borrowed capital must balance out positive properties of debt, following from the existence of tax shelter and its negative properties consisting in increase of bankruptcy risk as a share of debt capital increases.

In consequent investigations in the middle of 1970th other mechanisms were revealed that could influence the firm’s equity to debt ratio. These mechanisms arise from managers – equityholders and equityholders – debtholders conflicts of interests and information asymmetry of these parties and first described in Jensen and Meckling (1976), Ross (1977), Leland and Pyle (1977). The mechanisms are discussed in literature mainly on a qualitative level as determinants of corporate capital structure.

After forty years of research of the problem relevant literature includes hundred of titles and contains many interesting and useful findings, though not in all its directions achievements are equally sound. We would like to cite the opinion expressed by Hayne E. Leland in his Presidential Address to the participants of fifty-eighth annual meeting of the American Finance Association, Leland (1998).

“Financial economics has a rich literature analyzing the capital structure decision in qualitative terms. But it has provided relatively little specific guidance. In contrast with the precision offered by the Black and Scholes option pricing model and its extensions, the theory addressing capital structure remains distressingly imprecise. This has limited its application to corporate decision making”.

The opinion is authoritative, because its author himself develops one of few fruitful approaches in the area of quantitative assessments of optimal capital structure, (Leland (1994, 1998), Leland and Toft (1996)).

According to the approach, the dynamics of corporate (share) value is represented as a diffusion process in continuous time. This allows derivation of the
partial differential equations for a market value of a firm, market value of a corporate
debt and various functions of a firm's value. The use of the diffusion model allows
obtaining in some cases closed form decisions and tracing the dependence of the
optimal capital structure on various parameters used in the model. This is the
remarkable advantage of the approach. A good survey of the related studies one can

At the same time the approach has some drawbacks. The most important in our
opinion consists in fact that the obtained conclusions concerning optimal capital
structure are insufficiently firm-specific.

Let us clarify this statement.

The diffusion approach like other tax-based approaches is based on the fact
that the amount of debt a firm can use is limited by the increasing probability of its
bankruptcy. Clearly, a recommendation concerning the choice of an optimal capital
structure must be highly specific for a firm, its business, its current financial position
and parameters of debt (cost of service, maturity horizon etc). Authors investigating
the problem of bankruptcy prediction identify the set fundamental factors influencing
the possibility of corporate default:

- corporate profitability, current and cumulative (retained earnings position),
  operational activity, stability of earnings, liquidity, working capital position, interest
  coverage, size of a corporation – in Altman’s studies of 1968 and 1977 (see
  Altman (1982));
- various cash flow variables relative to total assets - in Aziz, Emanuel and
  Lawson’s (1988) model;
- market return variables - in Clark and Weinstein’s (1983) model.

Altman (1982) identified also a set of macroeconomic parameters influencing
the probability of corporate bankruptcy.

Being tightly linked with the probability of corporate bankruptcy optimal Debt /
Equity ratio must depend on the same set of factors.

There are too few possibilities to account for the factors in the diffusion model.
Diffusion model determines optimal capital structure in dependence on the value of a
corporation and its volatility only. The recommendations appear to be too common.
The results are in some (unknown) way averaged with respect of other influencing
factors.
The current paper proceeds with the study of the problem of optimization of corporate capital structure within the probabilistic Bayesian approach described in Philosophov L., Philosophov V. (1999). The main idea of the approach consists in direct use of estimates of probabilities of a firm’s bankruptcy in future in calculation of its value. To determine optimal amount of debt, one must first assess probabilities of a firm’s bankruptcy at various moments in future (in dependence on the amount of debt, other indices of a firm's current financial position and macroeconomic factors) and then calculate how these probabilities influence the corporate (share) value.

The above study was based on Altman's – type prognosis to some degree adjusted to accounting for time remaining until the bankruptcy. The incentive for the new investigation appears due to the new issues in probabilistic bankruptcy prognosis presented in Philosophov L., Philosophov V. (2002).

The authors present qualitatively different approach to bankruptcy prognosis from that described in Altman (1980), though both studies are based of prognostic factors derived from accounting information (balance sheets and income statements).

To ensure independent reading of this paper in its second section we shall shortly summarize outcomes of Philosophov L., Philosophov V. (2002) study with emphasize on these aspects of the problem that are important for capital structure issues.

In the preceding section one we shall discuss some details of formulation of the problem of optimization corporate capital structure that are important for a subsequent study.

In the third section we shall present a systematic description of our findings concerning the conditions of existence and value of optimal debt to equity ratio.


Tax advantage connected with the use of debt financing form one of the main factors, predetermining the existence of the optimal capital structure. When ratio Total Debt / Stockholders Equity is small its increase leads to proportional increase of a firm’s value because of tax shelter effect. Further increase of share of debt capital increases probability of a firm’s bankruptcy and thus reduces its value. At some amount of Debt / Equity ratio negative effect of debt balances out its positive effect. At this moment the value of the firm achieves its maximum.
Assume that financial position of a firm at some time moment $t_0$ (moment of making capital structure decision) is described by a set (vector) of factors (indices) $f$. Assume also that set $f$ includes as well factors that determine value of a firm at time $t_0$, so that the value appears to be a function of $f$, $V = V(f)$. Possible bankruptcy of a firm at some time moment $t_b$ in future also influences its value. To emphasize this important in context of the study dependence one can write $V = V(f, t_b)$.

Among many methods of assessment a firm's value, its dependence on time interval, when a firm is going concern, is best of all reflected by discount methods. Bankruptcy in this case means drying up of cash flow, or flow of net profits, or dividend payments. In the simplest case the firm's value with accounting for the restricted time of its being going concern can be expressed as:

$$V(f, t_b) = \sum_{i=1}^{i_{t_b}} E_i(f) \left(1 + d\right)^{-i},$$

where $E_i(f)$ is firm's net income in $i$ - th year.

Decrease of a firm's value caused by its bankruptcy is usually expressed in literature in form of bankruptcy costs – missed gains, losses that a firm bears in times preceding its bankruptcy and expenses directly connected with the legal bankruptcy procedures. Altman (1984) determines these components as indirect and direct cost of bankruptcy. The cost of bankruptcy is usually represented as a share ($\delta$) of firm's value lost in bankruptcy.

To account for the cost of bankruptcy one must add to the expression (1) one more component

$$V(f, t_b) = \sum_{i=1}^{i_{t_b}} E_i(f) \left(1 + d\right)^{-i} + (1 - \delta) \cdot \sum_{i_{t_b+1}} E_i(f) \left(1 + d\right)^{-i},$$

reflecting a share of a firm's value, retained by equityholders after the firm's bankruptcy.

There are several papers Altman (1984), Andrade, Kaplan (1998) that estimate direct and indirect costs of bankruptcy quantitatively on the base of empirical data. Using these estimators with formula (2) one must note that decrease of a firm's profits and losses that the firm suffers in times immediately preceding the bankruptcy
are usually included in coefficient $\delta$ as indirect cost of bankruptcy. Hence, they must not influence the values of $E_i(f)$.

In this case $E_i(f)$ must correspond to profits that the firm could have long before its bankruptcy or be assessed from the profits of other firms of the same line of business at the same time period.

Because of random character of possible bankruptcy event and moment $t_b$, firm’s value $V(f, t_b)$ appears also to be random. As optimal one must consider capital structure that maximizes $\bar{V}(f)$ - mean value of $V(f, t_b)$:

$$\bar{V}(f) = \int V(f, t_b) \cdot P(t_b / f) \cdot dt_b,$$

(4)

where $P(t_b / f)$ is conditional probability of firm’s bankruptcy at time $t_b$ given its financial position at time $t_0$ is described by vector of factors $f$.

Would the expression for conditional probability $P(t_b / f)$ be fully known, its substitution into (1) could give on principle possibility to determine the form of dependence $\bar{V}(f)$. Among components of vector $f$ in this case one could find factor characterizing the degree of financial leverage of a firm. Maximization of $\bar{V}(f)$ by this factor could enable finding optimal proportion of stockholders’ own and borrowed capital and determining its dependence on factors (components of $f$) characterizing other aspects of a firm’s financial position.

We shall suppose that conditional probabilities $P(t_b / f)$ do objectively exist and can be (theoretically) determined by means of mental experiment, where one takes a large ensemble of identical firms with equal starting financial position $f$ at time $t_0$, and lets them work free and independently of each other. Calculating the share of firms becoming bankrupt during each time interval $t_i, t_{i+1}$ one can assess probability density $P(t_b / f)$.

Repeating this experiment for a variety of possible firms’ initial financial positions $f$, one could determine how these positions influence probability of a firm’s bankruptcy and what particular factors are of importance.

Another way of thinking consists in supposition that trying to assess optimal capital structure one can make first a probabilistic prognosis of a firm’s bankruptcy at different future times by means of Bayes formula:
\[
P(t_b / f) = \frac{P(f / t_b) \cdot P_{Ap}(t_b)}{\int P(f / t_b) \cdot P_{Ap}(t_b) \, dt_b},
\]  

(5)

where \( P_{Ap}(t_b) \) is prior probability density of firm's bankruptcy at time \( t_b \), and \( P(f / t_b) \) likelihood function – joint conditional distribution of prognostic factors \( f \) at initial moment \( t_0 \) for firm, that becomes bankrupt at the moment \( t_b \).

The prognostic probabilities can be then used for calculation \( \bar{V}(f) \) by means of (4).

The second way is much more realistic, but has also an inherent drawback. This consist in fact that while in first case a set of factors influencing probability of bankruptcy is determined internally “within the problem”, in the second case it is usually determined externally in a separate investigation not connected with capital structure problem.

Furthermore, different authors propose different sets of factors to solve the same prognostic problem. This brings some uncertainty concerning true determinants of optimal corporate capital structure and optimal Debt / Equity ratio itself. At the same time optimal Debt / Equity ratio is an objective value that must not depend on a subjective choice of prognostic factors.

The solution of this contradiction consist in acknowledging the fact that each prognostic rule can give some estimator of the set of influencing factors together with estimation of optimal Debt / Equity ratio. The more efficient is prognostic rule the more accurate will be estimator and the more accurately optimal Debt / Equity ratio can be assessed.

At the same time being to some degree superfluous the system of financial ratios admits alternative variants of description of a firm’s debt burden and its ability to pay the debt. Such variants can be to some degree equivalent and have equal predictive efficiency, though they can be differently accustomed to users.

The above considerations served as incentive for additional study of the problem of bankruptcy prognosis including prognosis of time horizons of bankruptcy. Findings of this study are represented in Philosophov L. Philosophov V. (2002). To ensure independent reading of this paper we shall give a summary of the study in next section.

2. Prediction of corporate bankruptcy within a specified time interval.
To make simultaneous probabilistic prognosis of the bankruptcy event and time interval of its possible occurrence the prognostic problem was considered as a multialternative problem of the Statistical Decision Theory.

As competing were considered hypotheses $T_i$ of a firm's bankruptcy during $1, 2, \ldots, i, \ldots$ year starting from the moment of observation.

As empirical information was used data of USA firms, that became bankrupt between 1980 and 1988. Information block for each firm included up to seven balance sheets and income statements for last seven years before a firm's bankruptcy.

To choose informative prognostic factors the authors have traced evolution of cumulative sample distributions of pretending indices as a firm approaches its bankruptcy.

As competing were studied various factors that can be derived from accounting information - a firms' balance sheets and income statements. Among them were simple factors – various financial ratios and more complex like different modifications of Z-score rule from Altman (1982).

The comparative analysis of pretending factors gave possibility to choose four financial ratios with a remarkable prognostic potential and relatively independent. Two of them characterize the quantity and quality of corporate debt, while two other – a firms ability to pay the debt.

Factors of the second group are quite traditional and used as component parts of Z-score rule of Altman.

These are $f_3$ - \textit{Earnings Before Interest and Tax / Total Assets} (EBIT/TA) и $f_2$ - \textit{Retained Earnings / Total Assets}, (RE/TA), characterizing current and accumulated profitability of a firm.

Factor $f_4$ - \textit{Interest / Total Assets} (Int/TA) from the first group is to some degree equivalent to the familiar and proposed in literature factor \textit{Net Income / Interest} (Times Interest Earned). One can consider that it characterizes the amount of corporate debt and its quality (cost of service).

Replacing in $f_4$ Net Income by Total Assets one can reduce its dependence on the factor $f_3$ from the second group. This simplifies a prognostic procedure and increases its efficiency. In addition $f_4$ also fairly characterizes the cost of debt service in comparison with a firm's profitability.
The remaining factor from the first group $f_1$ - *Current Liabilities / Total Assets* (CL/TA), is perhaps the most interesting. According to our investigation a firm’s propensity to bankruptcy is determined not by amount of its total debt but rather by the amount of current liabilities, which usually drastically increase last year before a firm’s bankruptcy. In many cases this occurs because of approaching maturity time of large portions of long term debt that transforms into current liabilities within last year prior maturity. Some times this becomes the immediate cause of bankruptcy. At the same time a long-term debt can even decrease last year prior a firm’s bankruptcy.

In this way optimal ratio of corporate own and borrowed capital appears to be dependent on a structure of borrowed capital the share of long-term and short-term liabilities as well as on the maturity horizons of a long-term debt. One can consider maturity time to be another characteristic of the quality of long term debt.

Multialternative prognostic rules include calculation of posterior probabilities of above hypotheses $T_i$ and of various derivative hypotheses (bankruptcy within first two years, three years etc) To make decisions probabilities are compared with appropriate thresholds.

Efficiency of the prognosis is as follows:

- Events $T_1, T_7$ are predicted efficiently;
- Event $T_2$ can be predicted as $T_2$ or $T_1$ or $T_3$;
- Events $T_3 - T_6$ are hardly distinguishable from each other and from $T_7$.
- Derivative two-alternative prognoses of a firm’s bankruptcy during next one, two, three years are more efficient than Altman’s Z-score and known LR rules.

The analysis of empirical data evidences that for each of four above prognostic factors a rather distinct area of values exists that is typical only for concerns in prebankrupt state.

If values of all four prognostic factors are outside these areas, posterior probability $P(T_{1-6} / f) = P(T_1 / f) + ... + P(T_6 / f)$ of a firm's bankruptcy within nearest six years appears to be near to zero, while the probability $P(T_7 / f) = 1 - P(T_{1-6} / f)$ of bankruptcy in more distant perspective - near to one. This occurs because of large prior probability of the hypothesis $T_7$ (bankruptcy within the seventh year or later) given realistic prior bankruptcy rates 0,5% - 1% per year.
If some of these factors fall inside correspondent area, probability $P(T_{1-6} / f)$ abruptly increases, while $P(T_{7+} / f)$ falls. Value of a firm sharply decreases.

This effect is illustrated by figure 1 that represents dependencies of posterior probabilities of a firm's bankruptcy (denoted by letters 1, 2...6, 7+) on the values of factor $\text{Current Liabilities / Total Assets (CL/TA)}$.

One can see that within very narrow area of values of the factor $0.45<\text{CL/TA}<0.6$ posterior probability decreases from values $\sim 0.9$ until almost zero. Big values of probability $P(T_{7+} / f)$ when ratio CL/TA is small can be explained by big prior probability of hypothesis $T_{7+}$ while its abrupt decrease when CL/TA>0.5 is caused by the fact that such values of the factor are typical for firms in pre bankrupt condition.

![Figure 1. Posterior probabilities of a firm's bankruptcy $P(T_i / f)$, $P(T_2 / f),...P(T_{7+} / f)$ in dependence on values of the prognostic factor Current Liabilities / Total Assets (CL/TA). One can note abrupt decrease of the probability $P(T_{7+} / f)$ of a firm's bankruptcy within the seventh year or later when the factor value increases from 0.45 to 0.6.](image-url)
In figure 2 we represent the same dependencies for prognostic factor Interest / Total Assets (Int/TA). One can see that in this case too in relatively narrow area of factor values 0.075<Int/TA<0.085 probability $P(T_{7+}/f)$ abruptly decreases from almost one to almost zero.

Analogous situation can be observed for the factors $f_2$ - Retained Earnings / Total Assets, (RE/TA) and $f_3$ - Earnings Before Interest and Tax / Total Assets (EBIT/TA). For both factors border values are near to zero; posterior probability $P(T_{7+}/f)$ begins abruptly decrease when current and accumulated profits turn into losses.

![Figure 2](image)

Figure 2. Posterior probabilities of a firm's bankruptcy $P(T_1/f)$, $P(T_2/f)$,...,$P(T_{7+}/f)$ in dependence on values of the prognostic factor - Interest / Total Assets (Int/TA). One can note abrupt decrease of the probability $P(T_{7+}/f)$ of a firm's bankruptcy within the seventh year or later when the factor value increases from 0,075 to 0,085.

Figures 1,2 are obtained by means of the rules (algorithms) of bankruptcy prognosis described in Philosophov L., Philosophov V. (2002). Algorithms are
designed as a PC programs which calculate probabilities $P(T_1 / f), \ldots, P(T_6 / f)$, $P(T_7 / f)$ after inputting values of prognostic factors $f$. Analytic forms of dependencies of the posterior probabilities on values of prognostic factors are also obtained but they are rather complex and depend on many parameters.

We describe these findings in details because they are important for the current study.

3. Terms of existence and values of optimal debt to equity ratio.

Though when saying of optimal capital structure one usually considers optimal ratio of a firm's total debt to stockholders equity, a firm (if it is a going concern) must observe some proportions between long-term debt and current liabilities and then between various kinds of its liabilities (as well as assets, in which a debt capital is invested). As a result, any change in ratio of a firm's borrowed and own capital will be accompanied by a change of other factors characterizing firm's financial position which in turn will influence optimal choice of Debt / Equity ratio.

Following to established tradition we shall consider Total Debt / Stockholders Equity to be the principal ratio subject to optimization while another factors of a firm's financial position as parameters influencing that optimal value.

To determine conditions of existence of optimal Debt / Equity ratio and to assess its value we calculated a firm's value in dependence on that ratio. Formula (4) was used for these calculations while posterior bankruptcy probabilities $P(t_b / f)$ were determined by means of above mentioned algorithms of bankruptcy prognosis.

In our model value of stockholders equity was fixed and total debt relative to stockholders equity was considered as independent variable.

Of four factors influencing a firm's bankruptcy prognosis two factors $f_1$ - Current Liabilities / Total Assets and $f_4$ - Interest / Total Assets are immediately dependent on firm's capital structure and change as the structure changes.

Factor $f_2$ - Retained Earnings / Total Assets also changes (decreases) as firm's assets increase, though this dependence can be more complex. On one hand the factor characterizes accumulated amount of firm's net earnings relative to its assets and hence decreases as the assets increase. On the other hand the factor characterizes positive experience of a firm's activities in past and skill of its managers. In this respect it must not decrease as assets of a firm increase. In current
study we will mainly consider that the factor does not change as the amount of debt changes.

Other factors do not depend on a firm's capital structure but nevertheless do characterize its financial position and through that position the existence and value of optimal Debt / Equity ratio. Among them is prognostic factor \( f_3 \) - \( EBIT / Total \) Assets and some other factors, influencing a firm's value, including factors of macroeconomic environment.

Increase of borrowed relative to stockholders own capital leads to simultaneous increase of factors \( f_1 \) and \( f_4 \). Until any of these factors does not achieve its critical value, probability of a firm's bankruptcy in near years is small and firm's value increases as total debt increases.

When one of factors achieves and then exceeds its critical value probability \( P(T_{T\alpha} / f) \) begins drastically decrease. In this moment value of a firm achieves its maximum, corresponding to optimal ratio of Total debt / Stockholders equity. Depending on parameters of debt the critical value is first achieved either by factor \( f_1 \) or by \( f_4 \).

Factor \( f_1 \) - Current Liabilities / Total Assets (CL/TA) can achieve critical value \( \sim 0.5 \) if more than half of new debt is formed by current liabilities.

This can be seen from formula:

\[
CL/TA = CL / (EQ + LTD + CL),
\]

where: \( EQ \) denotes stockholders equity and \( LTD \) - long term debt.

Relation \( CL/TA > 0.5 \) holds if

\[
\frac{CL}{TD} > 0.5 \cdot \left( \frac{s + 1}{s} \right),
\]

where \( s \) is maximal acceptable value of Total Debt / Stockholders Equity ratio.

Dependencies of a firm's value on Debt / Equity ratio calculated at different shares of long term component \( dLD \) in total debt are presented in figure 3. Note that we speak now on long term debt with infinite maturity horizon. Such debt in context of this paper differs from equity only by a necessity of paying interest. If interest does not achieve its critical value this difference appears to be insignificant. The figure confirms above statement that optimal Debt / Equity ratio exists only if share of long term component of new debt is small, less than 0.4 in this example.
While calculating dependencies in figure 3 we set ratio $RE/TA$ to be equal 0.18 and interest rate on debt ($rIN$) - 8.8%. These figures are average in our empirical sample for firms long before their bankruptcy.

We have supposed also that discount rate ($rDC$) is equal to 10%, income tax ($rTX$) is 35% and a firm's profitability ratio ($EBIT/TA$) - 0.169.

Prior annual bankruptcy rate was set as 1% (this corresponds to statistical data of Dun&Bradstreet corporation).

With such values of above indices Net Income / Total Assets ratio of a firm will be equal to 11% (discount plus prior bankruptcy rate). A value of unlevered firm determined by discount method (with accounting for bankruptcy risk) will be near to value of stockholders equity. This creates a suitable starting point for assessing value of a firm that incurs debt.

![Figure 3. A firm's value in dependence on Total Debt / Stockholders Equity ratio. Curves in the figure correspond to different shares of short-term component in total debt (Current Liabilities to Total Debt ratio). More intensive increase of a firm's value when $CL/TD$ is high is due to lesser interest payments in this case. ($EQ = 1.00; RE/TA = 0.18; rBN = 0.01; EBIT/TA = 0.169; rTX = 0.35; rIN = 0.088; rDC = 0.1 ; kLQ = 1.0 ;)$]
It was supposed also that (relative) bankruptcy cost is equal to 1, i.e. stockholders loose in bankruptcy all their equity. Estimations of real bankruptcy costs based on empirical data one can find in several studies, for example, in already mentioned Altman (1984), Andrade, Kaplan (1998). The estimations show that bankruptcy cost is in average about 10 - 20 percents of firm's assets and is subject to significant variations from one case to another.

For current study loss ($kLQ$) of stockholders equity is of importance that exceeds above 10 - 20 percents because in bankruptcy stockholders' claims are satisfied in the last turn. In our sample value of stockholders equity last year before a firm's bankruptcy was about 0,175 of total debt. One can easily calculate that loss of 10 - 20 percents of firm's assets transforms into loss of 65 - 100 percents of stockholders equity.

Influence of bankruptcy cost on a firm's value, optimal Debt / Equity ratio and on gains in firm's value which optimization can give is evidenced by figure 4. Other parameters are held the same as in figure 3.
Figure 4. Dependence of a firm’s value on Total Debt / Stockholders Equity ratio at different levels \(kLQ\) of losses in bankruptcy of stockholders equity. If losses are zero bankruptcy does not influence a firm’s value \((EQ = 1.00; \ \text{RE} / \text{TA} = 0.18; \ \text{rBN} = 0.01; \ \text{EBIT/TA} = 0.154; \ \text{rTX} = 0.35; \ \text{rIN} = 0.088; \ \text{rDC} = 0.1; \ \text{dLD} = 0.2)\)

Curves in figure 4 correspond to values of relative bankruptcy cost \((kLQ)\), shown on the right-hand side of each curve. Curve for \(kLQ = 1\) in figure 4 is the same as curve for \(dLD = 0.2\) in figure 3. One can see from the figure 4 that finite optimal Debt / Equity ratio appears if stockholders loose in bankruptcy more than 50 percent of equity. If losses are zero bankruptcy does not influence a firm’s value that increases linearly as Debt / Equity ratio increases. Dependence of optimal Debt / Equity ratio and maximal value of a firm on relative bankruptcy cost is weak.

In general the above condition of existence of optimal capital structure

\[
\frac{\text{CL}}{\text{TD}} > 0.5 \cdot \left(\frac{s + 1}{s}\right)
\]

is rather rigid. In our sample when firm is far from bankruptcy (going concern) this ratio varies between 0.13 and 0.44 to be in average 0.27. The situation can change if debt has short maturity horizon. Last year prior maturity long-term debt transforms into current liabilities and \(\text{CL}/\text{TA}\) ratio can sharply increase.

Dependence of a firm’s value on Debt / Equity ratio for debt with finite maturity horizons is presented in figure 5.

While calculating dependencies in the figure 5 we set 44 percent of a firm’s total debt to be current liabilities and 56 percent - long term debt (these are average proportions in our sample). After some time denoted (in years) on the right hand side of each curve a firm returns 60 percent of long-term debt and then either restores initial capital structure or becomes bankrupt.

One can see in figure 5 that at small Debt / Equity ratios \((\text{TD/​EQ} < 1.5\) in current numerical example) approaching of maturity time does not lead to crucial increase of current liabilities and a firm’s value increases linearly as debt value increases. If Debt / Equity ratio exceeds 1.5, factor \(f\), \((\text{Current Liabilities} / \text{Total Assets})\) last year prior debt maturity achieves critical value, probability of a firm’s bankruptcy sharply increases and value of a firm achieves its maximum and then decreases.
Figure 5. Value of a firm in dependence on Total Debt / Stockholders Equity ratio for debt with different maturity horizons.

\( EQ = 1.00; \ RE/TA = 0.18; \ rBN = 0.01; \ EBIT/TA = 0.169; \ rTX = 0.35; \ rIN = 0.088; \ rDC = 0.1; \ dLD = 0.56; \ kLQ = 1.0 \)

On the right hand side of the maximum, value of a firm is mainly determined by discounted cash flows that firm can accumulate before long term debt matures. The value is the more, the more distant is debt maturity and for very long maturity horizons increases almost linearly as Debt / Equity value increases.

In the above cases maximally acceptable debt value was determined by a crucial increase of current liabilities immediately or around maturity time of long term debt. If share of current liabilities in total debt is small and maturity time of long term component is distant, crucial value of factor \( f_1 \) can stay unachieved. In this case another mechanism of a firm's bankruptcy can be important - when factor \( f_4 \) - Interest / Total Assets ratio exceeds its critical value.

In figure 6 we represent data that characterize dependency of optimal capital structure on a cost of debt (debt interest rate - rIN). Share of long term component in total debt was chosen rather high (~70%) so that critical value of factor \( f_1 \). (Current
Liabilities / Total Assets) is not achieved. Maximum of a firm’s value is obtained when value of the factor $f_d$ is near to critical $\sim 0.075$. One can see in the figure 6 that optimal capital structure exists if cost of debt is high ($\text{Interest} / \text{Long Term Debt} > 0.13$).

Probability of achievement its critical value by the ratio $IN/TA$ is higher when share of long-term debt in total debt is higher and higher is cost of debt. An exact conditions can be represented as an inequality that puts restrictions on the share of short term (and hence long term) debt in total debt of a firm.

The condition is as follows:

$$CL / TD < 1 - \frac{(IN / TA)_m}{rIN} \cdot \left(\frac{s + 1}{s}\right), \quad (7)$$

where $(IN / TA)_m$ – is critical value of ratio $IN/TA$ (it is equal to $0.075$); $rIN$ is interest rate on long term debt, $s$ – as before is maximal considered value of $TD/EQ$ ratio.

Figure 6. Value of a firm in dependence on Total Debt / Stockholders Equity ratio for debt with different interest rates (ratios Interest / Long Term Debt)

($EQ = 1.00; RE/TA = 0.18; rBN = 0.01; EBIT/TA = 0.169; rTX = 0.35; rIN = 0.088; rDC = 0.1; dLD = 0.70; kLQ = 1.0$)
In contrast with factors $f_1$, $f_4$, two other factors $f_2$, $f_3$ do not change as Debt / Equity ratio changes.

In figure 7 we represent dependence of optimal capital structure on factor $f_3$ ($EBIT / Total Assets$ ratio). It is supposed that $f_3$ (Gross return on assets) does not depend on amount of debt that firm employs. This factor influences a firm's value in two ways - via bankruptcy prognosis and via cash flows. It is supposed that after seven years counted out of date of observation firm must return 60 percent of long term debt.

One can see that dependence of optimal Debt / Equity ratio on value of factor $f_3$ is weak; though dependence of a firm's value when capital structure is optimal is more noticeable.

Figure 7. Value of a firm in dependence on Total Debt / Stockholders Equity ratio for firms with different profitability (ratio $EBIT / Total Assets$) ($EQ = 1.00$; $RE/TA = 0.183$; $rBN = 0.01$; $rTX = 0.35$; $rIN = 0.088$; $rDC = 0.1$; $dLD = 0.56$; $kLQ = 1.0$; Maturity Year = 7)
Figure 8 represents dependence of optimal capital structure on factor $f_2$ - Retained Earnings / Total Assets. According to discussion above we consider this factor to be also independent on amount of debt that a firm additionally incurs.

In this case also dependence of optimal Debt / Equity ratio on value of factor $f_2$ is also weak; while a firm's value when capital structure is optimal is influenced more significantly. This influence is more noticeable when value of the factor is near to critical $RE / TA \approx 0$.

Figure 8. Value of a firm in dependence on Total Debt / Stockholders Equity ratio for firms with different values of accumulated profitability (ratio Retained Earnings / Total Assets) ($EQ = 1.00; rBN = 0.01; EBIT/TA = 0.169; rTX = 0.35; rIN = 0.088; rDC = 0.1; dLD=0.56; kLQ = 1.0; Maturity Year = 7$)

In figure 9 we represent dependencies of a firm's value on prior annual bankruptcy rate. The rate characterizes prior probability (frequency) of corporate bankruptcies in country (industry) in dependence of macroeconomic situation:
economic growth activity; money supply; capital market activity; new business formation rate as Altman (1982) established.

The lesser is prior bankruptcy rate, the greater is amount of debt capital a firm can use and the more is gain in value it can achieve when Debt / Equity ratio is optimal. If annual bankruptcy rate is zero (i.e. a bankruptcy is impossible) one obtains classic linear dependence of Modigliani and Millier between a firm's value and Debt / Equity ratio.

Figure 9. Value of a firm in dependence on Total Debt / Stockholders Equity ratio for different values of prior bankruptcy rate (rBN).

\[ EQ = 1.00; RE/TA = 0.183; EBIT/TA = 0.169; rTX = 0.35; rIN = 0.088; rDC = 0.1; dLD = 0.56; kLQ = 1.0; \text{Maturity Year} = 7; \text{Share of maturing debt} = 0.6; \]

In addition to already considered factors influencing a firm's bankruptcy our model accounts for income tax and discount rates. These rates are considered here as some constants and their influence on optimal Debt / Equity ratio is not discussed.

To summarize in figure 10 in coordinates rIN, CL/TD we represent four areas restricted by condition (7) and above formulated condition (6). The borders of these areas are approximate and can shift slightly in dependence on other influencing factors.
Area I is typical for going concerns. It is characterized by small share of current liabilities in total debt and relatively low interest rates on long term debt. No one critical value is achieved. If for such a firm one considers change of *Debt / Equity* ratio to optimize a firm's capital structure, situation will depend on parameters of planned debt.

If parameters of a debt after change are again situated inside area I (figure 7) a firm's bankruptcy can occur only when large portions of principal long term debt mature (that shift a firm in the area II). Optimal capital structure can exist if maturity horizon is not distant – less than ten years. More often cause of bankruptcy in area I is unexpected and heavy current or accumulated losses, but it is only weakly dependent on capital structure.

Figure 10. Areas of values of factors *rIN* – interest rate on long term debt, *CL/TA* share of current liabilities in total debt of a firm with different mechanisms of existence of optimal capital structure (OCS).

In areas II, III one of critical values is achieved either by ratio *CL/TA* or by ratio *IN/TA* while in area IV both critical values are achieved simultaneously. In these
areas optimal ratio of a borrowed and stockholders own capital exists even for debt with infinite maturity horizon.

**Conclusion.**

In this paper we go on the study of the problem of optimization corporate capital structure within the approach proposed in Philosophov L., Philosophov V. (1999). The approach supposes the use of direct probabilistic prognosis of bankruptcy of a firm (in dependence of its current financial indices) together with a suitable model of a firm's value. The model must account for dependence of a firm's value on time interval of its being going concern before bankruptcy. Variants of discount cash flow model were used in both studies.

As it was supposed, additional investigation of the bankruptcy prediction problem carried out in Philosophov L., Philosophov V. (2002), resulted in more accurate definition of the set of factors influencing the bankruptcy prognosis and then optimal ratio borrowed to stockholders’ own capital.

The improvements are well explainable as that probability of a firm's bankruptcy must depend not only on Total Debt / Stockholders Equity ratio but also on such characteristics ("quality") of debt as cost of service and maturity horizons. It was revealed that a firm's short term and long term debt influence bankruptcy prognosis in different way.

Establishing of more exact set of prognostic factors gave possibility to create more detailed picture of influence of debt characteristics on existence and value of optimal capital structure. In one (and most probable) area of debt characteristics optimal capital structure can exist only for debt with short and middle (less than ten years) maturity horizons. In two other areas predominant factors are cost of debt and ratio of short term and long term components of debt. In these areas optimal capital structure exists also for debt with long maturity horizons.

At the same time we must emphasize (as it was explained above) that the proposed set of influencing factors gives a next (more exact) approximation of a true set that must jet be found.

In particular one of implicitly used suppositions is that of mutual independence of prognostic factors. This supposition is forced by restricted volume of sample of observational data at our disposal. This volume is insufficient to create rules of bankruptcy prognosis that take in account true mutual dependence of prognostic...
factors. More detailed investigations of bankruptcy prediction problem based on expanded samples of empirical data are still actual.

Above calculations and figures give the idea of factors influencing the optimal capital structure and of the character of this influence. The approach is based on real data of real firms and does not use superfluously formalized models. We believe it can be used in practical capital structure decisions though specific calculations must be fulfilled for each firm that needs such decisions.
REFERENCES


